RHIC-AP-6

THE EFFECTS OF RF NOISE IN THE RHIC

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Due to intra beam scattering the momentum spread of the bunch is increased and hence the bunch length. Thus the rf bucket becomes quite full and in the limit the boundaries are exceeded.

RF noise also causes a blow up of the bunch area. Thus if not controlled it too can reduce the luminosity lifetime. When we propose a ten hour lifetime due to the first effect only, then one must examine the effects of noise. This is particularly of interest for the RHIC since as will be pointed out below a large spread in synchrotron frequencies which will be the case for nearly full buckets contributes significantly to the dilution.

The growth of the beam has been shown to be governed by a diffusion equation ($\rho(x,t)dX$ is probability of finding a particle between X&$(X+dX)$ at time t)

$$\frac{\partial\rho(X,t)}{\partial t} = \frac{\partial}{\partial X}\left((XG_1 + X^2G_2)\frac{\partial\rho}{\partial X}\right)$$

where $X = r^2\ (4/\pi)\ B(r^2)$ for a sinusoidal RF voltage and $r^2 = \sin^2\hat{\Phi}/2$ with $\hat{\Phi}$ being the peak or maximum bunch halfwidth. Here X is $\propto$ to the action variable J of the motion.

One also has the quantities (< > indicate ensemble averages all noises having the same power density spectrum)

$$A_1 = \left\langle\frac{\Delta X}{\Delta t}\right\rangle,\ A_2 = \left\langle\frac{(\Delta X)^2}{\Delta t}\right\rangle \text{ where } A_1 = G_1+2XG_2$$

$$\frac{A_2}{2} = XG_1 + X^2G_2 \qquad \omega_s = \omega_{so}\frac{\pi}{2K(r^2)}$$

with

$$G_1 = \frac{\omega_{so}^{\;2}}{4} \, \frac{2K}{\pi} \left(1-r^2\alpha(r^2)\right) \frac{\displaystyle\sum_{\ell=1,3,5\ldots}^{\infty} \frac{\ell^4}{\cosh^2(\ell\nu)} S_\phi(\ell\omega_s)}{\displaystyle\sum_{\ell=1,3,5\ldots}^{\infty} \frac{\ell^4}{\cosh^2(\ell\nu)}}$$

$$\nu = \frac{\pi}{2} \, \frac{K'(r^2)}{K(r^2)}$$

$$G_2 = \frac{\omega_{so}^{\;2}}{4} \, \frac{2K}{\pi} \, \frac{\pi}{4B(r^2)} \, \alpha(r^2) \frac{\displaystyle\sum_{\ell=2,4,6}^{\infty} \frac{m^4}{\sinh^2(\ell\nu)} S_a(m\omega_s)}{\displaystyle\sum_{\ell=2,4,6}^{\infty} \frac{m^4}{\sinh^2(m\nu)}}$$

Here B, K, K' are elliptic functions and α contains elliptic functions. $S_\phi(\omega)$ is the spectral power density of the random variable $\phi(t)$ and S_a the same for the variable a(t) where the rf voltage is represented by

$V = V_o(1+a)\sin\Phi$ pure amplitude noise

$V = V_o \sin(\Phi+\phi)$ pure phase noise

Now phase noise is much more harmful than amplitude noise so we will assume $G_2 = 0$ from here on.

Then we can write

$$\frac{dX}{dt} \approx \left\langle \frac{\Delta X}{\Delta t} \right\rangle = A_1 = G_1$$

where X is a measure of the area occupied by the beam and G_1 is some function of X.

We have calculated G_1 for two values of r^2 or $\hat{\Phi}$ namely $\Phi = 60°$ and $\Phi = 116°$. The latter is the value one obtains for a $\Delta p/p = 2 \times 10^{-3}$ in an RHIC bucket after two hours, for V_{rf} = 1MV and assuming the design intensities for Au at 100 GeV/A. The former represents a bucket that is 1/3 full (in bunch length) and is normally considered to be the maximum unperturbed area that can be stored for very long periods.

For given values of $S_\phi(\ell\omega_s)$ one could then calculate growth rates for X for these initial conditions. If one had explicit expressions for G_1 as a function of X he could in principal solve the diffusion equation with the boundary condition that $\rho(1,t) = 0$ and calculate a beam lifetime. This has been done for special cases none of which can be applied to the RHIC. Now the actual frequency noise seen by the beam arises from, and is affected by the low level rf system almost exclusively. Figures 1 & 2 taken from an SPS report show the feedback loops and indicate the sources of noise. Here P represents phase detector noise, F VCO noise, R frequency reference noise, $A(\omega)$ and $G(\omega)$ the phase and frequency loop transfer functions. (M is magnetic field error.) $B(\omega)$ is the beam transfer function for rf frequency errors. In the absence of frequency spread it is given by

$$B(\omega) = \frac{j\omega}{\omega^2 - \omega_s^2}$$

i.e. it has a pole at $\omega = \omega_s$.

For finite synchrotron frequency spread one must evaluate the dispersion integral. One then finds that $B^{-1}(\omega)$ £ S where S is the spread in synchrotron frequency. If we ignore magnetic field noise and solve the loop equations for y, and u where y is the frequency error seen by the beam and u is the output of the phase detector we can write

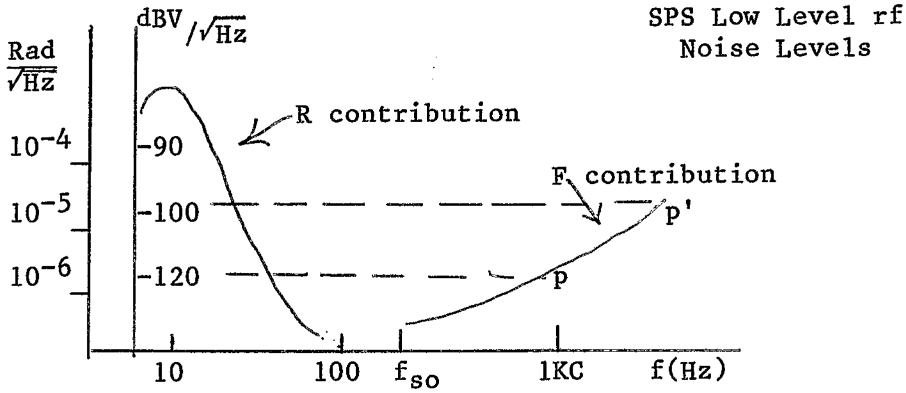
$$y = \frac{F + A(\omega)P + A(\omega)G(\omega)R}{1 + A(B(\omega)+G)}$$

$$u = \frac{B(F+AGR) + P + AGP}{1 + A(B+G)}$$

We are interested in the region around ω_s, $3\omega_s$ etc., where B>>G AB>>1 so that we can write y as

$$y = \frac{F}{AB} + \frac{GR}{B} + \frac{P}{B}$$

If one can choose A, G and ω_s along with finding a frequency reference & VCO so that F & R are quite small in the critical frequency range then the phase discriminator noise will dominate (see diagram below for the SPS) i.e., phase loop gain independent; only S & P determine lifetime



Then since $B^{-1} \backsim S$ where $S \simeq (\omega_{so}/16)\ \hat{\Phi}^2$ for $\hat{\Phi} < 1$ one can estimate y by assuming

$$y = \frac{P}{B} \simeq \frac{\omega_{so}}{16}\ \hat{\Phi}^2 P = \frac{\omega_{so}\ X\ P}{4}$$

and one can write

$$\langle y^2(\omega_s)\rangle = \frac{\omega_{so}{}^2\ X^2}{16}\ \langle P^2\rangle$$

For $r^2 = \sin^2 \hat{\Phi}/2 \ll 1$, G becomes just $\omega_{so}{}^2\ S_\phi(\omega)/4$. However we have $\langle y^2\rangle = \omega^2 S_\phi$ so finally for this simplified example

$$\frac{dX}{dt} = \frac{\omega_{so}^2}{64} X^2 \langle P^2 \rangle$$

Now for $X_o = 0.27$ i.e., for $\Phi = 60°$ $f_o = 180$ Hz(SPS) and $P = 1.75 \times 10^{-5}$ rad r.m.s one obtains a doubling time for X of

$$\tau_2 = \frac{64}{\omega_{so}^{\ 2}\langle P^2(\omega_s)\rangle X_o} = \frac{64 \times 10^{10}}{4\pi^2 .25 \ 1.75^2 \ 180^2} = 65\text{x}10^4 \text{sec}$$

or 180 hrs!! for the SPS and 2700 hrs for RHIC if $f_{so} = 46.4$ Hz. Really for $\hat{\Phi}_o = 60°$ we must use the complete expression for G_1. We find

$$G_1 = \frac{\omega_{so}^{\ 2}}{4}\left(.806 \ S_\phi(\omega_s) + .0218 \ S_\phi(3\omega_s)\right)$$

Next we assume

$$y(\omega\ ) = \frac{P(\omega_s)}{B(\omega_s)} \ ; \ y(3\omega_s) = \frac{P(3\omega_s)}{B(3\omega_s)}$$

We also assume $P(\omega_s) = P(3\omega_s)$ and have for $\hat{\Phi}/2 = 30°$, $B^{-1}(\omega_s) = .068\ \omega_{so}$ while $B^{-1}(3\omega_s) = 8\omega_{so}/3$; remembering that $\langle y^2(3\omega_s)\rangle = 9S_\phi(3\omega_s)$ we obtain

$$G_1 = \frac{\omega_{so}^{\ 2}}{4}\left(.806\text{x}.\,068^2 + \frac{64}{81}\text{x}.0218\right)\langle P(\omega_s)^2\rangle = \frac{\omega_{so}^{\ 2}}{4}\langle P^2\rangle\left(3.72\text{x}10^{-3}+17.2\text{x}10^{-3}\right)$$

That is the noise at $3\omega_s$ is almost five times as effective as that at ω_s!!

Again we compute the doubling time for the RHIC case where now

$$\tau_2 = \frac{X_o}{G_1} = \frac{4 \times .27}{\omega_{so}^{\ 2}\ 1.75^2\text{x}10^{-10} \times 20.9\text{x}10^{-3}} = 1.98 \times 10^6 \text{ sec}$$

$$= 550 \text{ Hrs}$$

Next we consider the case where $\hat{\Phi} = 116°$ or $X_o \cong 0.82$ which corresponds to the expected conditions after two hours for the Au bunches

We find

$$G_1 = \frac{\omega_{so}^2}{4}\left(.41\ S_\phi(\omega_s) + .24\ S_\phi(3\omega_s)\right)$$

This time $B^{-1}(\omega_s) = .256\ \omega_{so}$ and again $B^{-1}(3\omega_s) = 8\omega_{so}/3$. The term for $\ell = 5$ is still negligible here.

We obtain

$$G_1 = \frac{\omega_{so}^2}{4}\left(.0269 + .1897\right)\langle P^2\rangle$$

where now the $3\omega_s$ term is $\cong 7$ times larger than the ω_s term. Note also that the individual terms here are 7 and 11 times larger than for $\Phi = 60°$. Again we calculate τ_2

$$\tau_2 = \frac{4x.82x10^{10}}{\omega_{so}^2\ 1.75^2x.217} = 580x10^3\ \text{sec} = 161\ \text{Hrs.}$$

Now as a limiting case we consider $\hat{\Phi}/2 = 80°$, $r^2 = .97\ X_o = 1.203$ and assume that $P(\ell\omega_s)$ is flat for large ℓ and that it is the only noise present. Then we can remove it from the Σ's which then give unity. We have $K(r^2) = 3.15$ and $\alpha \cong 8/15$ which along with $P(\ell\omega_s) = 1.75 \times 10^{-5}$ rad/$\sqrt{\text{Hz}}$ enable us to calculate G_1 and hence to obtain τ_2 (note that $\omega_s(.97) = \omega_{so}/2$)

$$\tau_2 = \frac{4 \times 1.203}{\omega_o^2 x.9652x1.75^2x10^{-10}} = 19.1x10^4\ \text{sec} = 53\ \text{Hrs.}$$

Hence even with this large a bunch one would need $\cong 5$ hrs. for 10% additional growth. This however would result in some beam loss. It is of course a limiting case where we have assumed an order of magnitude larger value for $P(\omega)$ than now claimed by the SPS group.

In the above limit by removing S_ϕ from the summation and assuming $P(\ell\omega_s)$ is flat we have ignored any effects of the beam transfer function. The phase loop merely serves to keep the VCO noise below the phase detector noise while the frequency loop must not contribute noise over the incoherent frequency band.

Now we have not considered the effects of amplitude noise for such large bunches. Also we must consider the effects of noise at $f_{rf} \pm nf_o$ on all the other bunches in the ring i.e., not all the bunches see the same noise. Hence the phase lock loop which is closed on one bunch only takes care of noise around f_{rf}. This effect can be corrected by additional lower gain but wider bandwidth loops. It can be reduced by using high Q cavities whose bandwidth is $<f_o$ and could be eliminated for n = 1 by putting half the cavities on opposite sides of the ring.

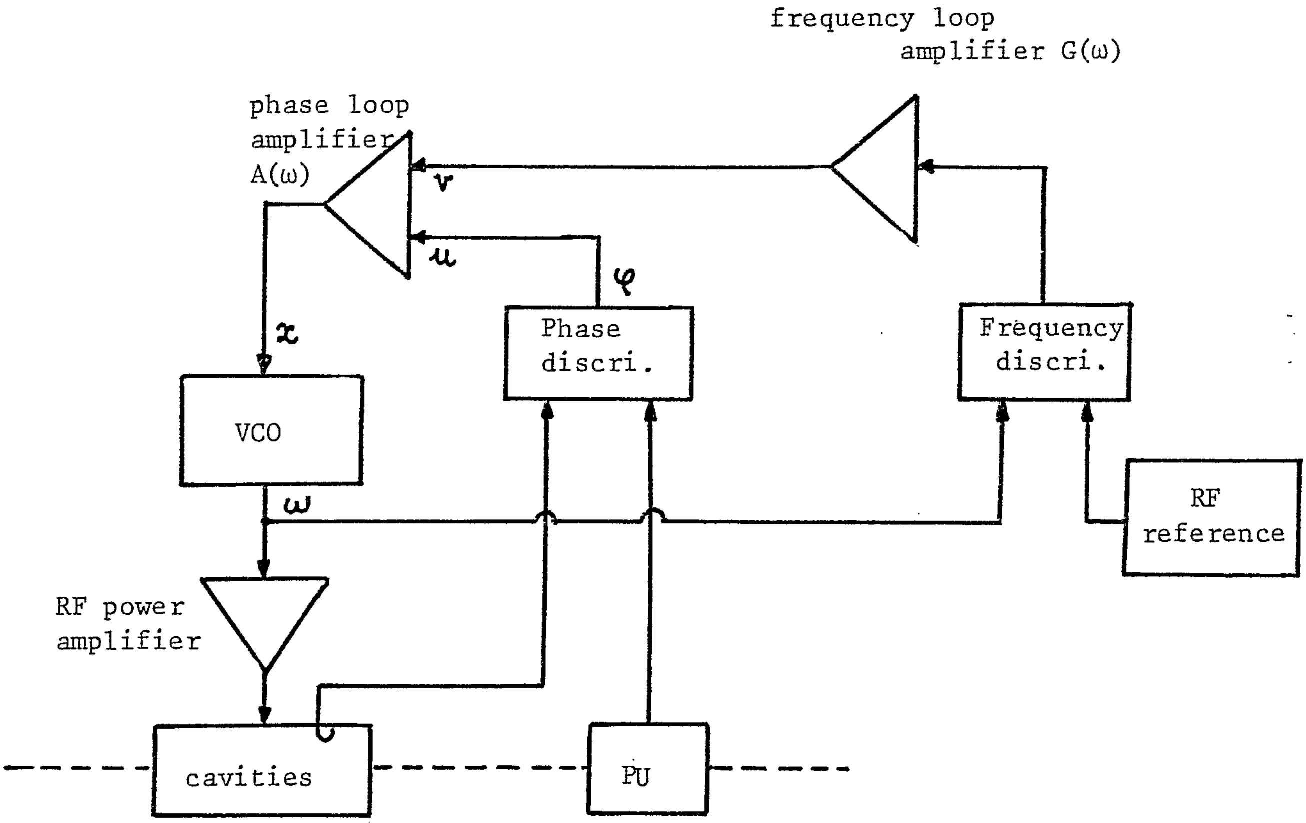


Fig. 1 - Schematics of RF loops

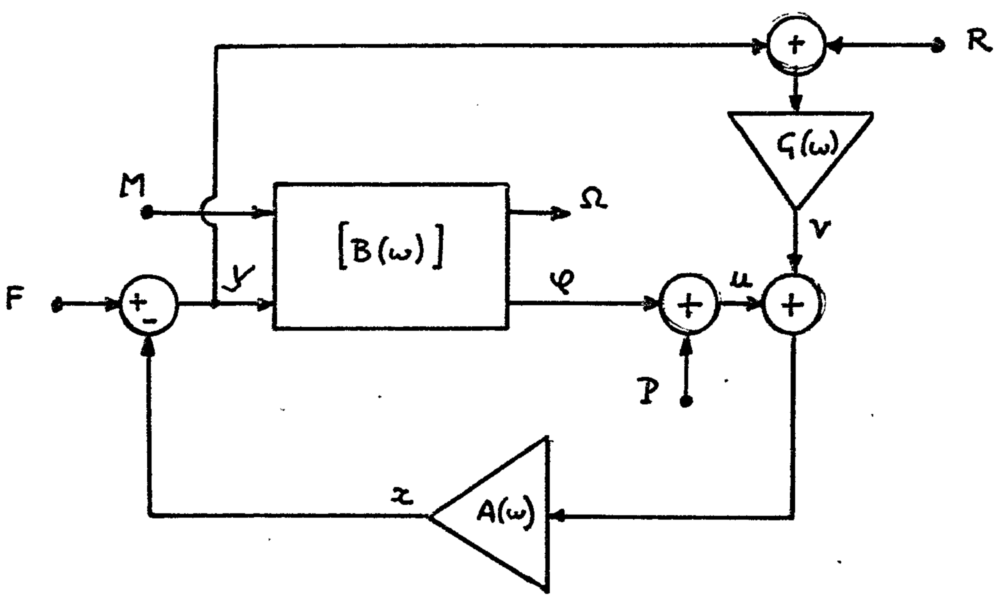


Fig. 2 - Equivalent circuit of RF loops
Noise sources: F, M, P, R.
Test points: x, u, v